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A new approximate expression for the response of a hot-wire anemometer

By MICHIO NISHIOKA AND MASAHITO ASAI
Department of Aeronautical Engineering, University of Osaka Prefecture, Sakai, Osaka, 591 Japan

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Bruun's (tabulated) universal function for the response of a hot-wire anemometer is examined in detail to see if the function has some unique relationship with King's law. As a result of the analysis, an approximate expression is derived and proposed for the response over a wide range of wind speeds. Although the expression is like King's law with an additional correction term and is thus quite simple in form, it agrees with Bruun's universal function (and also with the derivative, i.e. the velocity sensitivity) to within 3% over the range of wind speeds from 2 to 120 m/s. There is little doubt that the proposed response equation may serve as a helpful guide in calibrating a hot-wire probe, in constructing a simple linearizer circuit of high accuracy, and in linearization by means of a computer.

1. Introduction

The hot-wire anemometer has been one of the most reliable tools for measuring fluctuation velocities in turbulent flows. It is well known that our progress in understanding various turbulence phenomena has been based on the so-called hot-wire data obtained by numerous investigators. Most of them have contributed to its refinement into the present instrument as traced in the bibliography presented by Freymuth (1978). At least for the foreseeable future, we believe, the hot-wire anemometer will not lose its present key position in the field of experimental turbulence research, and any further improvement with respect to its accuracy and feasibility is desirable.

The operating principle of the anemometer is quite simple. It is based on the heat transfer from the sensing element, i.e. a fine wire (of tungsten or platinum, typically of 5 μm in diameter), which is electrically heated above the temperature of the flow to be measured. For the use of hot-wires in normal low-speed air-flow facilities, the pressure and the temperature are kept constant near standard atmospheric conditions. The maximum wind speed is limited to, say, 100 or 150 m/s at most. The length to diameter ratio (l/d) is fixed for a given hot wire. In most cases the wire temperature is kept constant through controlling the heating current by means of a negative-feedback circuit (the so-called constant-temperature mode). Under these conditions, and for wind speeds greater than 1 or 2 m/s, it can safely be said that the total heat loss is dominated by forced convection. Therefore, according to King (1914) and Collis & Williams (1959), we can relate the Nusselt number to the square of the voltage output (of the anemometer) E^2 as follows:

$$E^2 = A + B(U_{\text{eff}})^n, \tag{1}$$

where A, B and the exponent n are expected to be constant for a given wire, and U_{eff} is the effective cooling velocity. Several expressions have been proposed for the relation between U_{eff} and the velocity magnitude U. The most familiar among these is

$$U^2_{\text{eff}} = U^2 (\cos^2 \phi + k^2 \sin^2 \phi) \tag{2}$$

which is given by Hinze (1959) and Webster (1962): ϕ is the angle between the velocity vector and the normal to the wire. Champagne, Sleicher & Wehrmann (1967) determined k as a function of l/d. Therefore, the response equation for $\phi = 0$ (namely for the case where the hot wire is placed normal to the velocity vector),

$$E^2 = A + BU^n, \tag{3}$$

is of fundamental importance. King (1914) obtained the value of 0.5 for the exponent n, while Collis & Williams (1959) found $n = 0.45$ to fit to their calibration data for long wires ($l/d > 2000$).

In most practical anemometer applications, the exponent n is commonly taken as 0.5 (King's law) or 0.45 (Collis & Williams' law) and the constants A and B are determined through calibration measurements for each probe by plotting E^2 against U^n. It is also common practice to use a linearizer which generates an output voltage E_{L} proportional to the effective cooling velocity. This can be done quite easily, in particular for the case of these power laws. Unfortunately, however, laws with constants as described above are approximations only useful over a limited wind-speed range, normally from 1 to 15 m/s at most as will be illustrated later. In order to find a universal empirical heat transfer law applicable over a wide wind-speed range Bruun (1971*a*, 1976) made elaborate calibration measurements. He demonstrated that the calibration curves for all hot-wire probes of a give type (used with a fixed support orientation and a specified type of anemometer) can be approximated to a very high accuracy by the equation

$$E^2 - E_0^2 = Cf(U). \tag{4}$$

Here E_0 is the measured voltage output at $U = 0$, $f(U)$ a universal shape function for all such probes and C an individual calibration constant for each hot-wire probe. Instead of using analytical expressions, he tabulated the universal function $f(U)$ in order to avoid possible errors due to mathematical approximations. This means that it was difficult to find an accurate, simple analytical expression for the universal function, though Bruun (1971*b*) proposed

$$E^2 = a + bU^{\frac{1}{2}} + cU \tag{5}$$

for wind speeds ranging from 5 to 60 m/s. Equation (5) looks simple, but it is not particularly easy to construct a linearizer circuit for it.

In the present paper, we shall examine carefully Bruun's universal function in order to seek a simple expression for the response of a hot-wire anemometer.

2. Bruun's universal function and King's law

The sensing element of the hot-wire probes that Bruun (1976) calibrated was a 2 mm long 5 μm tungsten wire having a cold resistance (at room temperature) of the order of 7.0 to 7.5 Ω. The wire had copper-plated sheaths at both ends. The length of the prongs was about 8 mm, and the distance between them was 5 mm, while the diameter of the probe stem was 3 mm. Thus, the probe is of typical geometry for

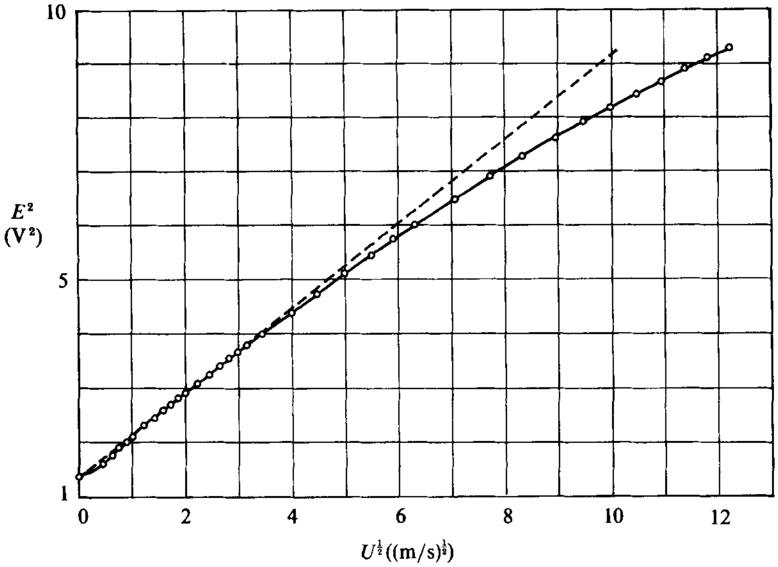

FIGURE 1. Comparison of Bruun's universal function (— ○ —) with King's law (---).

turbulence measurements. The stem was placed parallel to the mean flow, with the wire axis normal to the mean-flow direction. The operating-wire resistance was approximately twice the cold resistance. The calibration measurements were made using a constant-temperature anemometer.

Figure 1 illustrates Bruun's (1976) universal function (solid curve) by plotting E^2 against $U^{\frac{1}{2}}$ for a ready comparison with King's law (broken line). The constants A (in V^2) and B (in V^2 $(m/s)^{-\frac{1}{2}}$) of King's law are so determined that the broken line passes through calibration points at $U = 4$ and 8 m/s:

$$E^2 = 1.374+0.7664U^{\frac{1}{2}}. \tag{6}$$

As mentioned earlier, it is seen from figure 1 that King's law seems to deviate from the universal function beyond about 15 m/s and below about 1 m/s. The reason for the deviation below about 1 m/s is no doubt that the amount of heat loss owing to forced convection becomes smaller than losses due to conduction (to the prongs) and free convection. Possible low-Reynolds-number effects on the forced convection also come into play. Some detailed calibration measurements below 1 m/s were made by Nishioka & Sato (1974). On the other hand, it seems quite difficult to point out the possible causes for the deviation beyond about 15 m/s. In connection with the concept of a universal function, it is important to mention Perry's speculation. Perry (1982) states that because of aeroelastic deflections (of the sensing element), it seems unlikely that a universal functional form for the heat transfer exists and that each wire has its own individual characteristics. The thermal expansion (or elongation) of the sensing element might produce an elastic bow and the wire becomes a buckled column with built-in ends. If Bruun's probe had suffered from the aeroelastic deflections and related vibrations, the comparison in figure 1 would have no general meaning. Incidentally, the present authors always examine each hot-wire probe under a microscope, but have never seen such deflections or vibrations. Bruun carefully checked the reproducibility of the calibration measurements. Thus, we may assume that Bruun's data are also free from such problems. Nevertheless, we should

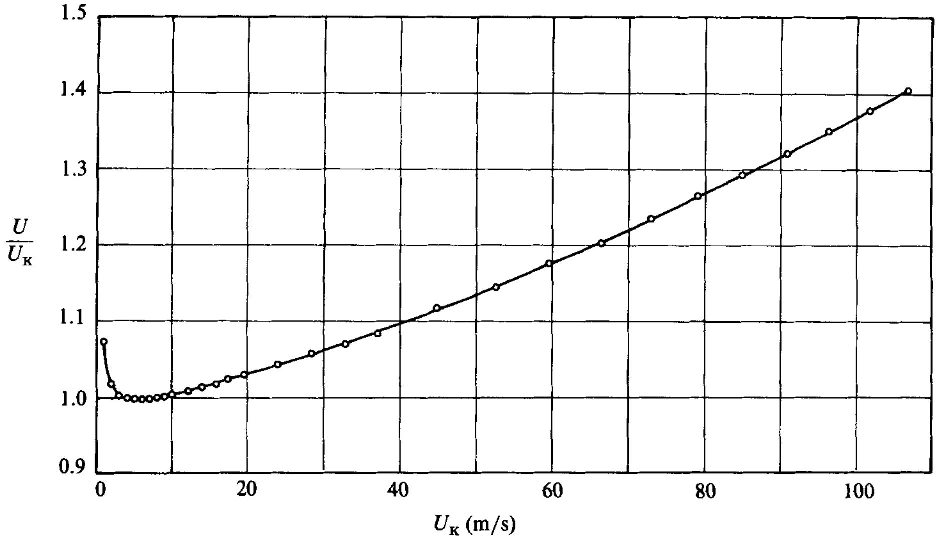

FIGURE 2. Bruun's universal function, plotted as U/U_K *vs.* U_K, where U_K is given by (6).

not forget the possibility that a small thermal elongation might have a 'strain gauge' effect on the wire resistance. Moreover, it should be noted that in any constant-temperature operation the wire temperature is not truly kept constant, but it decreases inevitably when the heating current is increased by means of the negative-feedback amplifier. Whilst such problems might have a significant effect on the hot-wire response, at the present time we do not know the actual cause for the deviation beyond 15 m/s. It is also important to emphasize that this deviation is not peculiar to Bruun's calibration only, but is widely known to hot-wire users, and of course reproducible.

3. New approximate expression for the hot-wire response

Using the same calibration data, we plot U/U_K against U_K in figure 2 to show the deviation more in detail, where U_K is the velocity calculated from (6) (King's law) for each voltage output E. In this figure the wind-speed range over which King's law holds closely is only between 3 and 10 m/s. Bruun's universal function deviates from King's law ($U/U_K = 1$) monotonically beyond 10 m/s. However, it is very interesting to note that the relation between U/U_K and U_K yields the following, simple analytical expression:

$$U/U_K = 1+\epsilon U_K^{\frac{3}{2}}, \tag{7}$$

$$\epsilon = 3.82\times 10^{-4}\ (\mathrm{m/s})^{-\frac{3}{2}}, \tag{8}$$

where the value of ϵ is determined so as to fit (7) to Bruun's calibration at $U = 60$ m/s. It is worth noting that U_K is a measure of the heat loss due to the forced convection. In this sense, the deviation appears to be a function of the heat loss by forced convection.

Figure 3 compares Bruun's universal function with (7) (with U_K given by (6)) by plotting E^2 against U. The abscissa is taken as a logarithmic scale in order to show possible relative errors (deviations) clearly. An excellent agreement is obtained between them over a wide range of wind speeds from 1 to 150 m/s. Generally

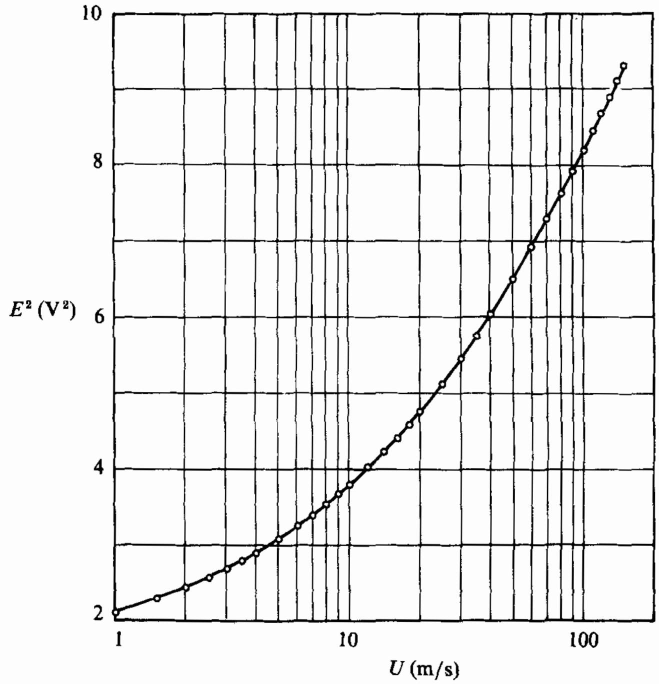

FIGURE 3. Bruun's universal function, plotted as E^2 *vs.* U. The solid line represents (7).

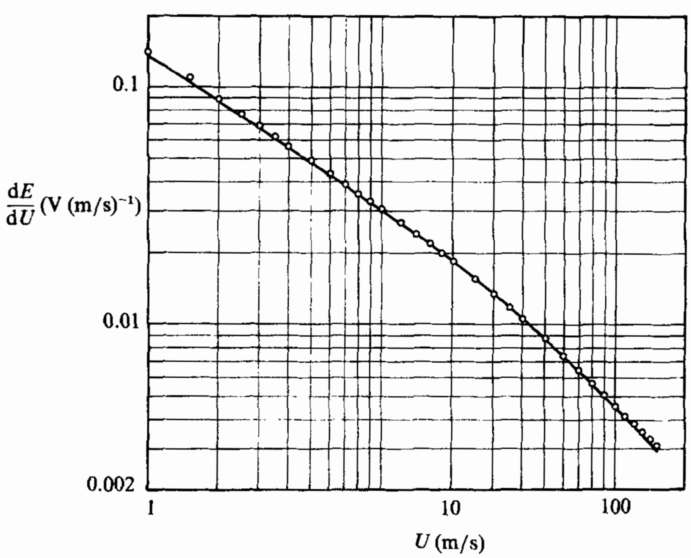

FIGURE 4. The velocity sensitivity of Bruun's universal function, plotted as dE/dU *vs.* U. The solid line represents (7).

speaking, however, a representation like figure 3 is insufficient to check the mutual agreement closely because the agreement might not be so good with respect to the velocity sensitivity dE/dU. So we present a comparison of the sensitivity in figure 4; also the ratios U/U_B and $(dE/dU)/(dE/dU)_B$ are given in figure 5, where the suffix B means Bruun's universal function. As clearly demonstrated in figure 5, over the

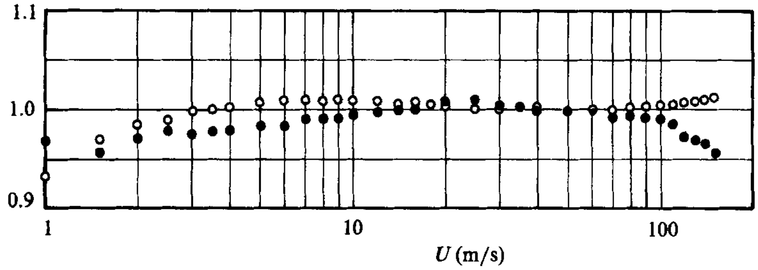

FIGURE 5. Detailed comparisons between Bruun's universal function and (7). ○, U/U_B; ●, $(dE/dU)/(dE/dU)_B$.

range of wind speeds from 2 to 120 m/s, both the velocity and the sensitivity from (7) agree with Bruun's calibration to within 3%. This is really excellent considering the wide wind-speed range over which (7) is applicable.

With all the results presented thus far, we may propose the following new expression for the response of a hot-wire anemometer:

$$U = U_K(1 + \epsilon U_K^m), \tag{9}$$

$$E^2 = A + BU_K^{\frac{1}{2}}, \tag{10}$$

where the constants A and B are determined over a low wind-speed range where King's law (10) holds closely, and the constants ϵ and m are determined so as to fit (9) to calibration data beyond the low-speed range mentioned above. The constants A, B and ϵ will vary from hot wire to hot wire. We speculate that the constant m will not vary so much, remaining near 1.5. It should be added that $m = 1$ and 2 (with proper values of ϵ) also work well for the case of Bruun's calibration, though not better than $m = 1.5$, in particular beyond 90 m/s.

Concerning linearizing the response to obtain the voltage output E_L proportional to U, the proposed expression is quite tractable since it is like King's law with an additional correction term, U_K^{m+1}; namely, by using an additional circuit for the correction term, the linearized output signals are obtained for the time-mean and fluctuation velocities. The new expression is also useful in linearization by means of a computer, and it is helpful in calibrating a hot wire over a wide range of wind speeds, as is King's law for a lower wind-speed range. In fact, its validity and usefulness have been confirmed for probes other than Bruun's for example, for probes of tungsten wire with diameter $d = 3.7$ µm, length $l = 1.0$ mm, and $d = 5.0$ µm, $l = 2.0$ mm (in both cases, the operating wire resistance is 1.5 times the cold resistance) over wind speeds up to 80 m/s, the maximum velocity examined (the calibration data for these probes are described by Tanaka 1974).

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